Ranking the Severity of Ocean Waves for Flexible Riser Design – Extreme and Fatigue

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1. Abstract

This paper addresses methods and case studies for ranking the severity of ocean waves to allow efficient design of dynamic flexible risers. The methodology is based on motions analysis of a flexible riser hang-off location on a floating platform. The types of ocean wave models considered include long-crested (regular and irregular) seas and short-crested irregular seas.

2. Introduction

Flexible risers are bending compliant pipes and umbilicals between subsea and surface topsides facilities. Risers convey produced, injected and hydraulic fluids and power and control lines. Dynamic flexible risers are deployed from floating platforms, mainly FPSOs and semisubmersibles. The majority of dynamic structural loading on flexible risers is caused by platform motion in ocean waves. These motions induce oscillatory loading along the full length of a flexible riser and are particularly significant near topsides and subsea interfaces.

This paper addresses methods and case studies for ranking the severity of ocean waves to allow efficient design of flexible risers. The methodology is based on motions analysis of a flexible riser hang-off location on a floating platform. The types of ocean wave models considered include long-crested (regular and irregular) seas and short-crested irregular seas. The analysis is also dependent on motion characteristics of the floating platform.

The ranking is applied to extreme and non-extreme wave conditions for extreme loading and establishing wave bins for fatigue loading. The ranking is used to prioritise an evaluation order of load cases to facilitate an efficient design procedure. The ranking can be applied at the concept, FEED or detailed design stages.

The methods described in this paper also apply to dynamic risers in general, including steel catenary risers, top-tensioned risers and hybrid risers. These riser systems are not specifically addressed in the paper.

3. Extreme Design Waves

The selection of design waves for (dynamic) flexible risers has its origins with fixed platforms where wave height and steepness are key considerations.

The selection of extreme and fatigue design waves for fixed platforms is relatively straightforward. Wave loading on a fixed platform generally increases with wave height and steepness.

Design waves for flexible risers have traditionally used wave height as the main selection criteria with limited sensitivity checks for wave period. Regular waves are normally used in the first instance for design, due to simpler analysis methods in comparison with irregular waves. The regular wave is based on the most probable maximum wave height (Hmax) during a sea state and this is specified with a small range of associated wave periods (Tass).

Methods for specifying Hmax in terms of sea state parameters are well established. The range of associated wave periods is less readily determined. The range is approximately within ten percent of the spectral-peak period (0.9 to 1.1 x Tp). A design can be sensitive to Tass near a resonant period of the platform motion (e.g. pitch or roll motion). The design in these cases applies a comparatively large regular motion (steady-state oscillation) on the floating platform which is uncharacteristic of the motion induced by an occurrence of a Hmax wave in an irregular sea. Adjusting Tass away from a resonant period reduces the sensitivity of the design. The extent of adjustment relies on a case-specific assessment to achieve an acceptable design. An alternative approach tests a potentially critical condition in an irregular sea simulation configured with a Hmax wave and short duration. The process is however less efficient for design.

Caution is also necessary in less extreme sea states (e.g. 10 year return period) with smaller Hmax and Tass is near resonance periods of the platform motion. These sea states have potential for inducing larger motions than a sea state with a larger Hmax but Tass further away from the resonance periods of the platform motion.

A riser design assesses several combinations of loading conditions for a floating platform and riser and this includes an extensive set of design waves. The process can become time consuming especially if riser analyses are conducted for each loading condition and associated sensitivities. Significant delay in identifying the most appropriate design waves can lead to improvised planning, impact on the project budget and reduced assurance of the selected design waves.

Combinations of loading conditions that specifically affect platform motions include:

- Wave height x period x direction
- Platform draught – ballast to fully loaded
- Platform heading – head, quartering, beam and stern waves

The method described below improves the efficiency of this process by screening the platform motions at the riser hang-off location with comparatively quick analyses. The method is readily implemented before commencing riser analysis. The method is demonstrated for analyses based on regular waves and wave spectra. The results of the analyses based on wave spectra are used to determine the wave period of a Hmax design wave. The method avoids excess sensitivity near resonance periods of the platform motion.

4. Riser Hang-Off Motion

The loading response of a flexible riser deployed from a floating platform is closely correlated with vertical oscillatory motion at its fixed hang-off location, e.g. riser turret or riser porch. Vertical motion at a hang-off location is a combination of platform heave, pitch and roll motions. The motion increases with wave height but dependencies on wave period and phasing of the component motions are more complex. Platform surge, sway and yaw motions produce horizontal oscillatory motion at a riser hang-off location, which is not usually significant for flexible riser design. The horizontal motion of platforms is more significant for steel catenary risers. Platform (horizontal) station keeping offsets is a further consideration that affects all riser types.

Riser system design for floating platforms has shown vertical motion at a riser hang-off provides a relative indicator on wave induced riser loading response, namely riser tension and bending. Vertical motion amplitude at a riser hang-off combines the effects of wave height, wave period, wave direction, platform draught and platform heading toward waves.

Riser hang-off heave for numerous design conditions can be efficiently evaluated before modelling the riser(s). Riser hang-off heave is evaluated from wave motion analysis of the platform response amplitude operators (RAOs). The RAOs define the amplitudes and phases of platform motion in response to a wave of unit amplitude and a range of wave periods (e.g. 3 to 30 s). The RAOs are normally specified for a central reference position on the platform. The motions at other positions on the platform are determined as part of a motions analysis. Vertical or heave motion at a riser hang-off is dependent on the platform RAOs for heave, pitch and roll and the position of the hang-off location relative to the RAO reference position. Riser hang-off heave is not always evident from inspection of platform motion RAOs.

5. Riser Hang-Off Heave – A Method for Extreme Design

Extreme design waves for flexible risers can be selected via analysis of the platform motions at the riser hang-off. A riser-hang-off motions analysis based on the platform RAOs is significantly more cost effective and less ad-hoc than detailed modelling of a riser to determine appropriate design waves.

A motions analysis can efficiently process the full range of extreme waves (heights, periods, directions) and platform conditions (draughts, headings). The riser-hang-off motion amplitudes can also be effectively plotted to provide a clear overview on ranking the severity of extreme waves for flexible riser design.

The motions analysis can be based on extreme regular waves (i.e. Hmax waves) or wave spectra of extreme sea states. The results of the wave spectra approach can be used to select an associated period (Tass) of a Hmax wave and avoid excess sensitivity to motions resonance as mentioned above in Section 3. See below for further details.

Regular Waves

The key wave parameters for a motions analysis based on regular waves are the most probable maximum wave height (Hmax) and the associated period (Tass). An individual Hmax wave has a range of Tass, which are often provided as lower, middle and upper values. The middle value is generally near the spectral-peak period (Tp). The lower and upper values are approximately within ±10 percent of the spectral-peak period (Tp).

The riser hang-off heave amplitude is based on the relationship:

h = ½ Hmax x RAO at Tass. (Eq. 1)

The RAO is platform response amplitude operator for vertical motion at the riser hang-off. The RAO is dependent on the platform motion RAOs and location of the riser-hang-off.

The heave amplitude normally refers to displacement. Heave velocity and acceleration amplitudes are also readily evaluated. The worst-cases of heave displacement, velocity and acceleration can be produced by different extreme wave or platform conditions. It is advisable to test the riser design for each of these worst-case hang-off motions. The maximum heave acceleration is sometimes used as an indicator for maximum riser tension and maximum heave velocity can indicate cases susceptible to axial compression if the riser has a low drag to weight ratio. Heave motion amplitudes are used to rank the severity of riser loading for the various combinations of wave and platform conditions.

The worst-case riser hang-off heave produced from a regular wave analysis can be overly conservative if Tass is near a resonance peak of the platform motions RAO. The conservatism can be reduced with selecting Tass based on results of a wave spectra analysis.

Irregular Seas – Wave Spectra

The key parameters for a wave spectrum of a sea state are significant wave height (Hs) and spectral-peak period (Tp). Other period parameters include the zero up-crossing period (Tz) and the spectral mean period (T1). Spectral bandwith and other shape parameters may also be specified.

The riser hang-off heave amplitude is derived from the heave motion spectrum:

Heave spectrum = Wave Spectrum x $|RAO|^2$, for all wave periods. (Eq. 2)

The RAO is for vertical motion at the riser hang-off as used for the regular wave motion analysis. The variance (σ_h^2) of the heave motion is the area under the heave spectrum. The standard deviation (σ_h) or significant amplitude ($2\sigma_h$) can be used to rank the severity of heave amplitudes for riser loading.

The most probable maximum heave amplitude is more effective for ranking the riser hang-off motion:

$h_{max} = \sigma_h \times (2 \ln(N))^{1/2}$, (Eq. 3)

N = sea state duration in seconds / Tz.

The sea state duration is nominally three hours in most cases and N is the number of zero up-crossing waves for this duration.

Revert to Regular Waves

Riser design simulations are normally computed with regular wave loading. The results for ranking riser hang-off heave amplitudes based on wave spectra can be converted to regular waves through using an associated wave period (Tass) for the most probable maximum wave height (Hmax) that produces the most probable maximum heave amplitude (h_{max}). The value for Tass is solved using the relationship:

h_{max} = ½ Hmax x RAO at Tass. (Eq. 4)

Tass is usually near T_p but not within strict bounds.

The process of selecting Tass ensures a riser analysis based on regular waves with a most probable maximum wave height constrains the riser hang-off heave motion to the most probable maximum heave and avoids excessive conservatism. Furthermore a motions analysis based on wave spectra produces wave height and period parameters suitable for riser design analysis based on regular waves.

Pairing h_{max} with Hmax via appropriate selection of Tass as described above can be implemented for the heave displacement, velocity and acceleration.

Experience based on time-domain riser analysis in irregular waves generally shows riser hang-off heave displacement is a reliable indicator of riser loading and it is not particularly necessary to rank hang-off motions for velocity and acceleration amplitudes. This comparison is less certain for analyses based on only regular waves and due caution with checking worst-case heave velocity and accelerations should be exercised.

6. Extreme Waves – Case Study

Results are presented for a turret moored FPSO with flexible production risers and umbilicals in 300 m water depth. The risers were designed in waves with 1 to 100 year return periods. The wave data was specified in terms of wave spectra (Hs, Tp, etc.), most probable maximum waves (Hmax, Tass – low to upper), in 16 directions equally spaced from North. The FPSO operates between ballast and fully loaded draughts and weathervanes towards head and bow-quartering waves (0 to +/- 45 degrees into waves). The FPSO motion RAOs were supplied as design data for the various draught and heading conditions.

The riser hang-off for the motion analysis was located on the port side of the turret. This ensures roll motions of the FPSO produce local heave motions at the riser hang-off in addition to FPSO heave and pitch. Results of the motions analysis based on regular waves and wave spectra are shown for all combinations of the wave and FPSO conditions.

Figure 1 shows a summary of results for riser hang-off heave amplitudes in regular waves. The results are shown on a radar plot with heave amplitude as the radial coordinate and direction waves ‘travel from’ as the angular coordinate (N, NNE, etc.). The heave amplitudes are normalised to show displacement, velocity and acceleration amplitudes on the same plot. The heave amplitude shown for each direction is the maximum resulting from combinations of wave and FPSO conditions applicable to that direction; 63 combinations per direction.

The riser hang-off displacement amplitude is clearly ranked highest for waves from NNE. This was reasonably evident from the design data as the largest sea states travel from NNE. Waves from NNW and WSW have the next highest rankings for displacement amplitudes. The plot can be used to select design waves in each riser loading sector (Near, Far and Cross). The wave producing the largest displacement amplitude in each sector would be selected as the design wave for the sector.

The riser hang-off velocity and acceleration amplitudes have highest ranked values for waves from NW and WNW, respectively. Highly ranked velocity amplitudes also occur for waves from NNW and NNE. Highly ranked acceleration amplitudes also occur for waves from NW and NE.

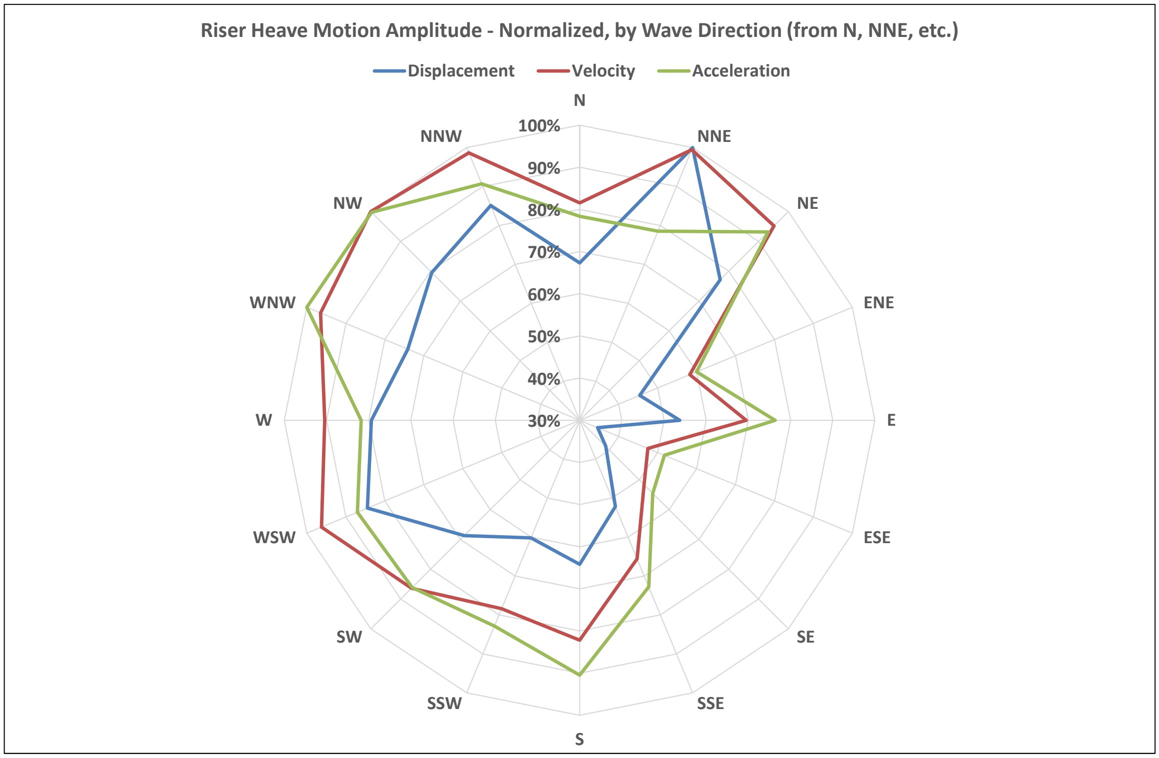


Figure 1: Extreme Waves – normalized heave amplitudes in Hmax regular waves

Figure 2 shows a summary of results for riser hang-off heave amplitudes using wave spectra. The plot layout is the same as Figure 1. Waves from NNE produce consistently high rankings for the heave displacement, velocity and acceleration amplitudes. There is also more consistency in the amplitude trends with changing wave direction. This consistency is helpful with selecting design waves in each riser loading sector (Near, Far and Cross). The motion amplitudes based on wave spectra are less sensitive to resonance periods in the motion RAOs of the FPSO.

Figure 3 to Figure 5 show more detailed results of heave amplitudes in regular waves. Results for waves from each direction include the platform draughts (3) and FPSO heading towards waves (7). Heave amplitudes for head waves are at midpoints of the curves for each draught. Quartering waves produce larger heave amplitudes than head waves in most cases. Waves from NNE are an exception where riser hang-off heave appears less sensitive to FPSO heading. Cases with the FPSO in head waves show waves from NNE consistently produce the maximum displacement, velocity and acceleration amplitudes. Head waves may provide an effective starting point for selecting design waves in each riser loading sector (Near, Far and Cross). Quartering waves test the robustness of the design, especially with waves from NW and WNW.

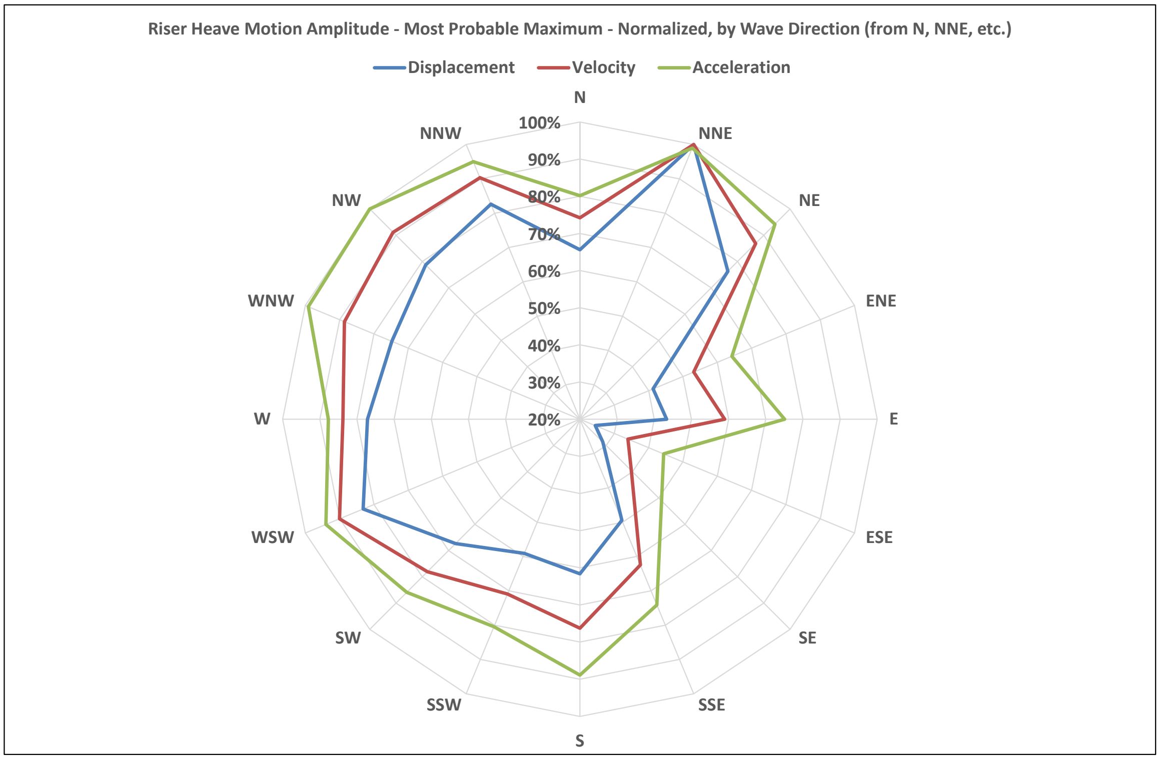


Figure 2: Extreme Waves – normalized heave amplitudes using wave spectra

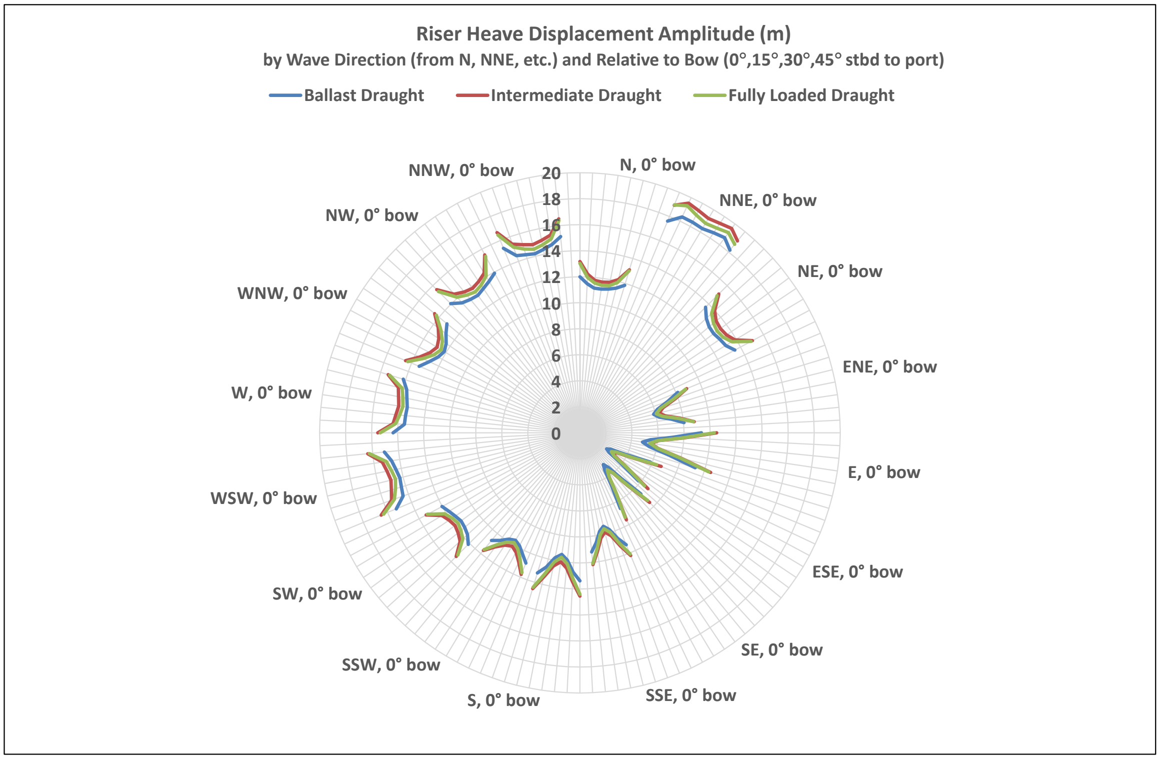


Figure 3: Extreme Waves – riser heave displacement in Hmax regular waves

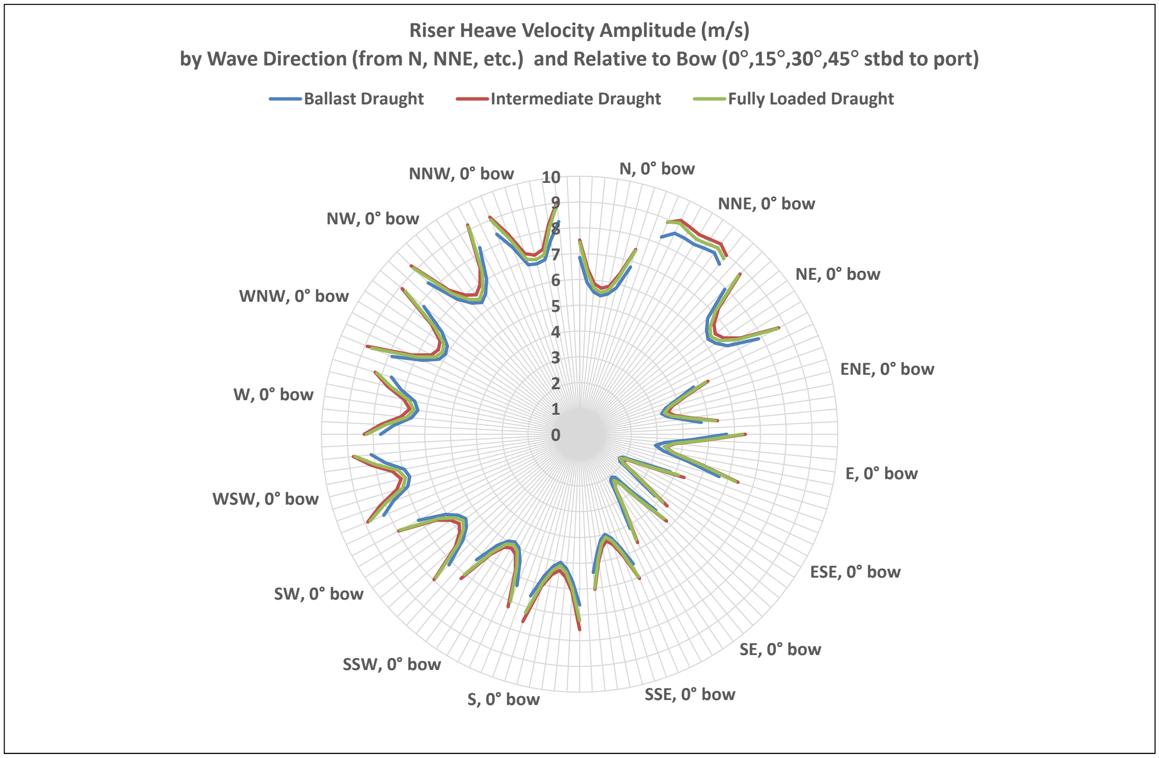


Figure 4: Extreme Waves – riser heave velocity in Hmax regular waves

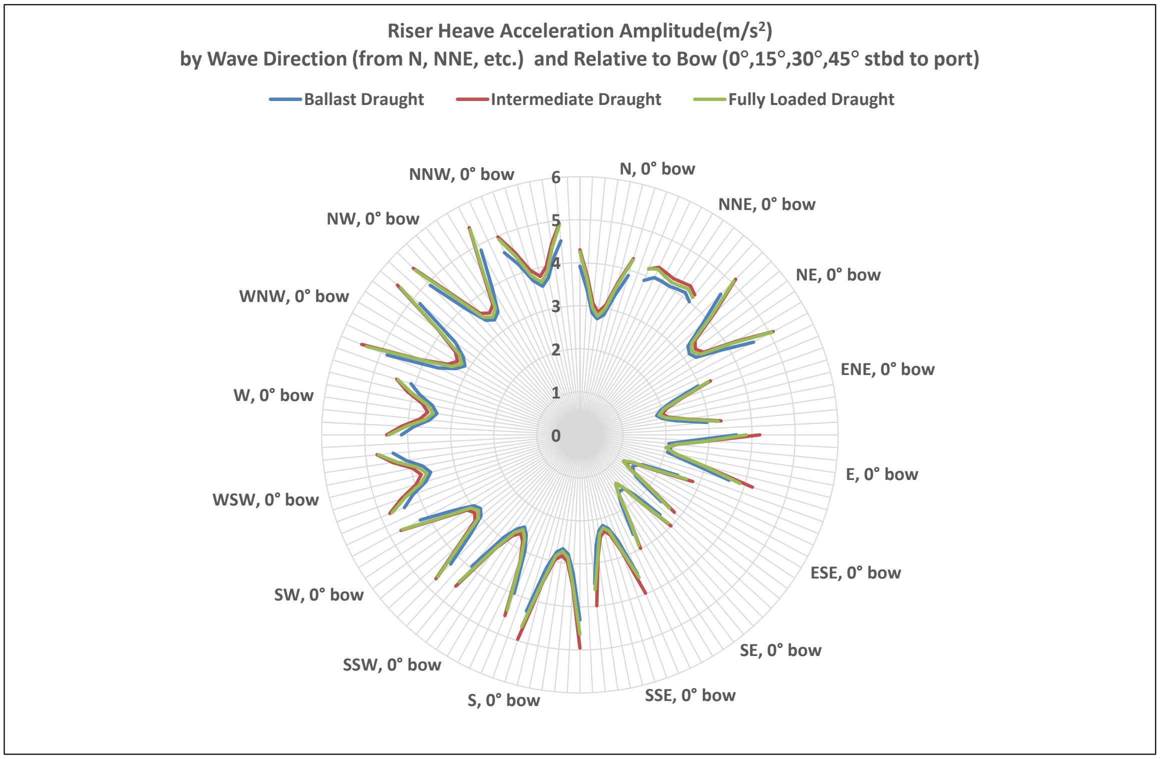


Figure 5: Extreme Waves – riser heave acceleration in Hmax regular waves

Figure 6 to Figure 8 show more detailed results of heave amplitudes using wave spectra. The plot layout is the same as Figure 3 to Figure 5. The heave amplitudes are the most-probable maximum for each condition plotted. The trends in the results are similar to Figure 3 to Figure 5 with two exceptions. First, the heave amplitudes are more sensitive to FPSO quartering headings with waves from NNE. Second and more importantly, the most-probable maximum heave amplitudes are significantly smaller than compared with heave amplitudes in regular waves.

Table 1 shows the worst-case heave amplitudes based on regular wave and wave spectra analyses. The amplitudes based on regular waves are between 18 and 38 percent larger than the amplitudes based on wave spectra.

Table 1: Riser Hang-Off Heave Amplitudes – Worst Case

Motion Type	Riser Hang-Off Heave Amplitude	
	Regular Wave[1]	Wave Spectra[2]
Displacement (m)	19.6	16.3
Velocity (m/s)	9.2	7.8
Acceleration (m/s^2)	5.5	4.0

1. Heave amplitude in regular waves based on Hmax and Tass.
2. Most probable maximum heave amplitude based on wave spectra.

The heave amplitudes derived from wave spectra can be implemented in a regular wave analysis by adjusting the associated wave period as described previously. Table 2 shows the associated period (Tass) of the required Hmax regular waves that produce the most-probable maximum heave amplitudes at the riser hang-off. The Tass values are within the nominal ten percent of the spectral peak period Tp, which is expected.

Table 2: Regular Wave Periods for Most-Probable Maximum Heave Amplitudes

Motion Type	Tass (s)	Tp (s)
Displacement	15.8	15.1
Velocity	14.5	15.1
Acceleration	12.2	11.9

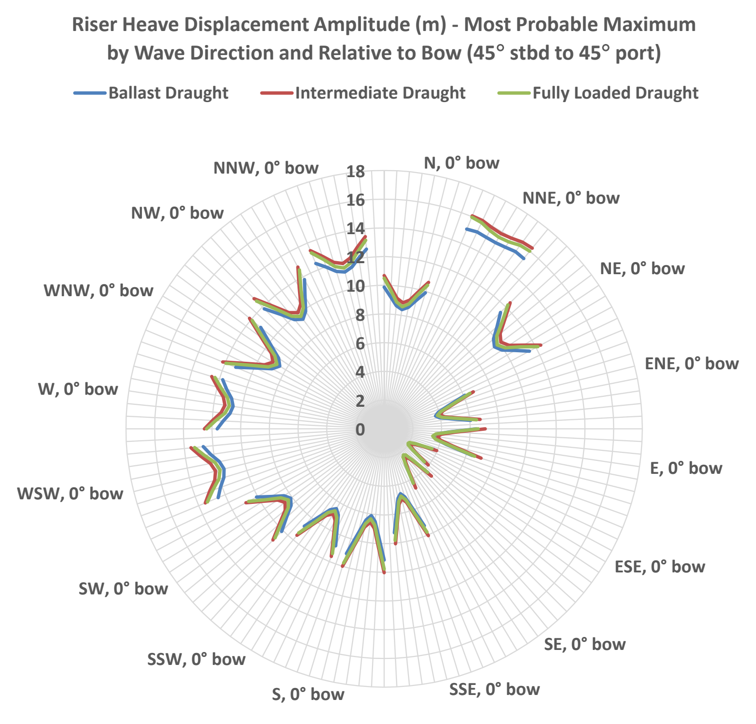


Figure 6: Extreme Waves – riser heave displacement using wave spectra

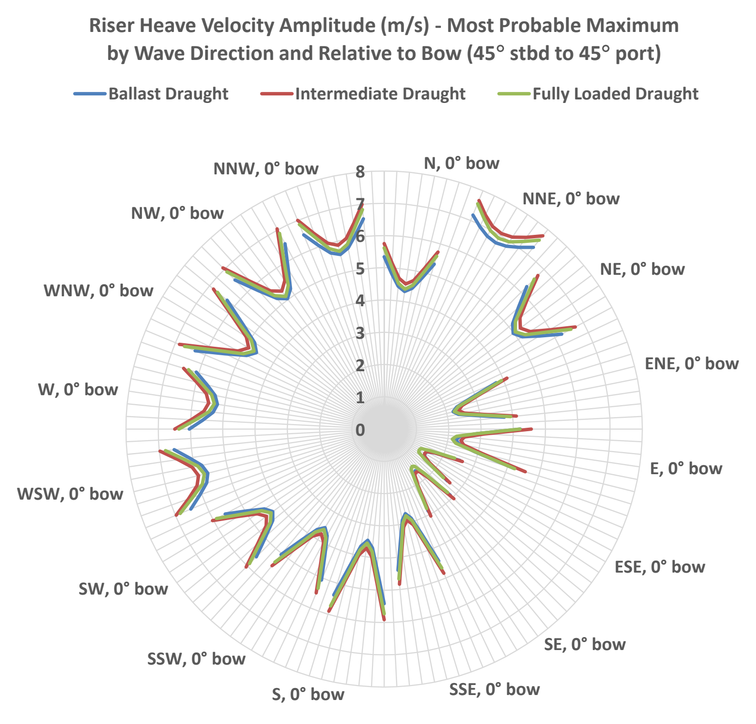


Figure 7: Extreme Waves – riser heave velocity using wave spectra

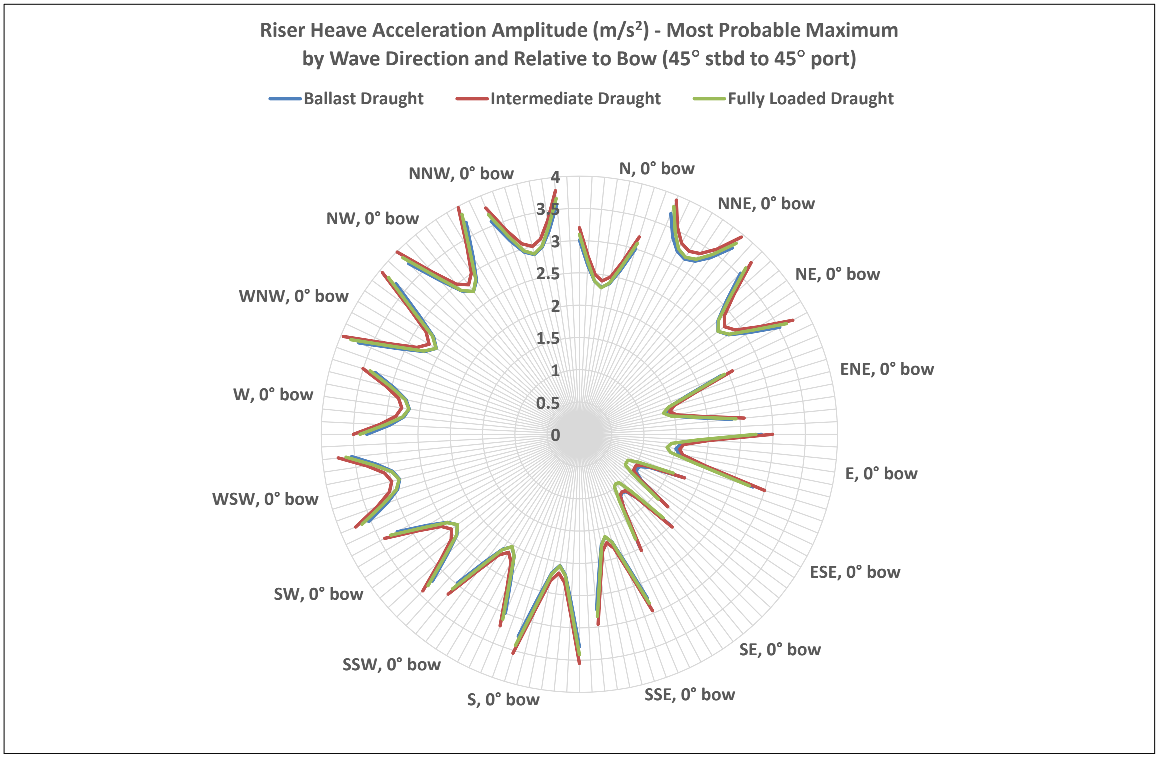


Figure 8: Extreme Waves – riser heave acceleration using wave spectra

7. Fatigue Design Waves

The fatigue loading of flexible risers is normally based on regular waves. Scatter tables for individual waves are the starting point for specifying regular wave heights and periods for a fatigue analysis. A scatter table may have more than 50 entries. A fatigue analysis normally reduces a scatter table to ten or less representative blocks to avoid unnecessary analysis and limit costs. The process is known as blocking a scatter table.

There are a number of design concerns with blocking a scatter table. The methods can be subjective and have limited formalised justification. A block can however be retrospectively subdivided where fatigue proves critical. The initial set-up and any retrospective change causes design uncertainty and inefficiency, which is compounded when multiple suppliers are involved.

A more formalised approach is described below where entries in the wave scatter table are converted to an estimate of relative fatigue damage. A brief description of current practice is presented first.

Figure 9 shows a simplified representation of an individual wave scatter table. It is effectively a two dimensional histogram table for the number of individual wave occurrences (per year) for paired bins of wave height and period, (wave bins).

Height (m)		Individual Wave Occurrences						
…				50	10			
4 – 5				400	50	2		
3 – 4			300	5000	300	30	1	
2 – 3		50	2.0E+4	3.0E+4	4.0E+3	150	15	
1 – 2		2.0E+4	3.0E+5	2.0E+5	3.0E+4	1000	200	10
0 – 1	6.0E+4	3.0E+6	2.5E+6	5.0E+5	6.0E+4	5000	3000	1000
Period (s)	0 - 2	2 – 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	…

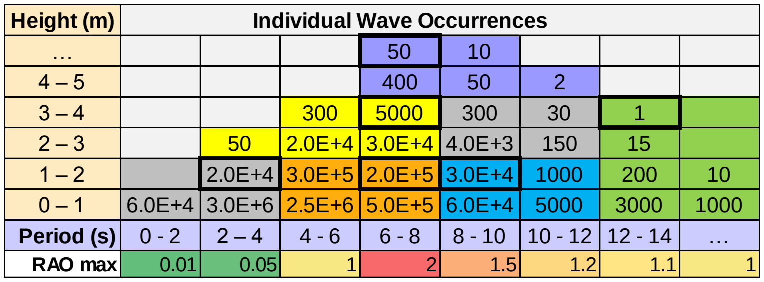
Figure 9: Individual wave scatter table – simplified

A basic method for blocking the scatter table to a smaller number of entries computes the mean wave period per wave height bin, weighted by the occurrences; see Figure 10. The number of occurrences (cycles) are summed for each wave height bin. The method is used for wave induced fatigue loading of fixed platforms and structures.

Height (m)	Period (s)	Cycles
…	7.3	60
4 – 5	7.2	452
3 – 4	7.0	5631
2 – 3	6.4	5.42E+4
1 – 2	5.9	5.51E+5
0 – 1	4.2	6.13E+6

Figure 10: Weighted average wave period per wave height

A second method accounts for the motion dependency of floating platforms on wave period via an RAO. The method define blocks by inspection of wave height vs platform motion RAO; see Figure 11. The blocked occurrences are colour coded and bold outline denotes the master wave. Occurrences in a block are summed. The master wave has the largest wave height and RAO value for the block. A platform pitch motion RAO might be used rather than a riser hang-off heave motion RAO.



Height (m)	Individual Wave Occurrences							
…				50	10			
4 – 5				400	50	2		
3 – 4			300	5000	300	30	1	
2 – 3		50	2.0E+4	3.0E+4	4.0E+3	150	15	
1 – 2		2.0E+4	3.0E+5	2.0E+5	3.0E+4	1000	200	10
0 – 1	6.0E+4	3.0E+6	2.5E+6	5.0E+5	6.0E+4	5000	3000	1000
Period (s)	0 - 2	2 – 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	…
RAO max	0.01	0.05	1	2	1.5	1.2	1.1	1

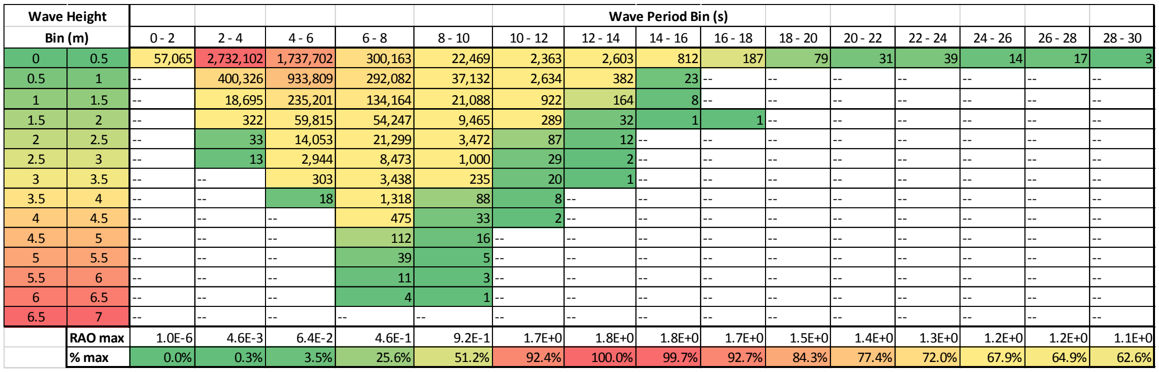
Figure 11: Blocks and master waves selected by wave height and RAO values

A more formalised approach converts the occurrences in a wave scatter table to an estimate of relative fatigue damage. The converted scatter table provides a fine grain estimate on the relative fatigue contribution of each wave bin. The occurrences are converted with the formulation:

(Wave Height x Riser Hang-Off Heave RAO)^n x (Occurrences) (%). (Eq. 5)

The exponent *n* = 1 is appropriate for ranking relative fatigue damage. The conversion is presented as a percentage of the overall total sum. Eq. 5 is also used for computing estimates of relative fatigue damage when blocking the scatter table.

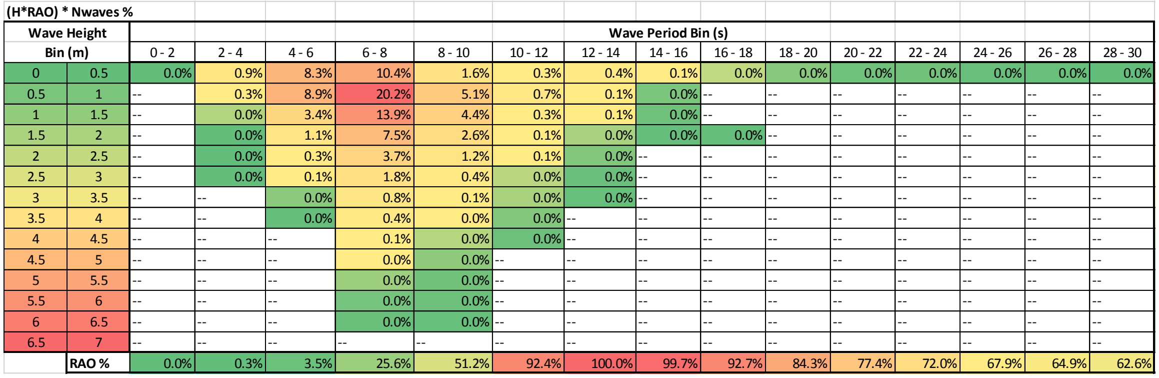
Results of the procedure are shown in Figure 12 to Figure 15. Figure 12 shows a more detailed wave scatter table. The bottom rows in the table shows the riser hang-off heave displacement RAO, including percentages of the maximum. The colour scheme on the table is scaled from low (green) to high (red) values. Higher fatigue damage is produced from wave occurrences somewhere in the yellow region rather than from high (red) wave heights, RAO values or occurrences.



Wave Height		Wave Period Bin (s)														
Bin (m)		0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20	20 - 22	22 - 24	24 - 26	26 - 28	28 - 30
0	0.5	57,065	2,732,102	1,737,702	300,163	22,469	2,363	2,603	812	187	79	31	39	14	17	3
0.5	1	--	400,326	933,809	292,082	37,132	2,634	382	23	--	--	--	--	--	--	--
1	1.5	--	18,695	235,201	134,164	21,088	922	164	8	--	--	--	--	--	--	--
1.5	2	--	322	59,815	54,247	9,465	289	32	1	1	--	--	--	--	--	--
2	2.5	--	33	14,053	21,299	3,472	87	12	--	--	--	--	--	--	--	--
2.5	3	--	13	2,944	8,473	1,000	29	2	--	--	--	--	--	--	--	--
3	3.5	--	--	303	3,438	235	20	1	--	--	--	--	--	--	--	--
3.5	4	--	--	18	1,318	88	8	--	--	--	--	--	--	--	--	--
4	4.5	--	--	--	475	33	2	--	--	--	--	--	--	--	--	--
4.5	5	--	--	--	112	16	--	--	--	--	--	--	--	--	--	--
5	5.5	--	--	--	39	5	--	--	--	--	--	--	--	--	--	--
5.5	6	--	--	--	11	3	--	--	--	--	--	--	--	--	--	--
6	6.5	--	--	--	4	1	--	--	--	--	--	--	--	--	--	--
6.5	7	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	RAO max	1.0E-6	4.6E-3	6.4E-2	4.6E-1	9.2E-1	1.7E+0	1.8E+0	1.8E+0	1.7E+0	1.5E+0	1.4E+0	1.3E+0	1.2E+0	1.2E+0	1.1E+0
	% max	0.0%	0.3%	3.5%	25.6%	51.2%	92.4%	100.0%	99.7%	92.7%	84.3%	77.4%	72.0%	67.9%	64.9%	62.6%

Figure 12: Scatter table of individual wave occurrences

Figure 13 shows the estimates of relative fatigue damage. The table shows the wave bins most likely to produce higher amounts of fatigue damage. These bins are difficult to identify by visual inspection of the scatter table for occurrences. Reduction of the scatter table to a small number of blocks can be implemented with more confidence on using the estimates of relative fatigue damage.



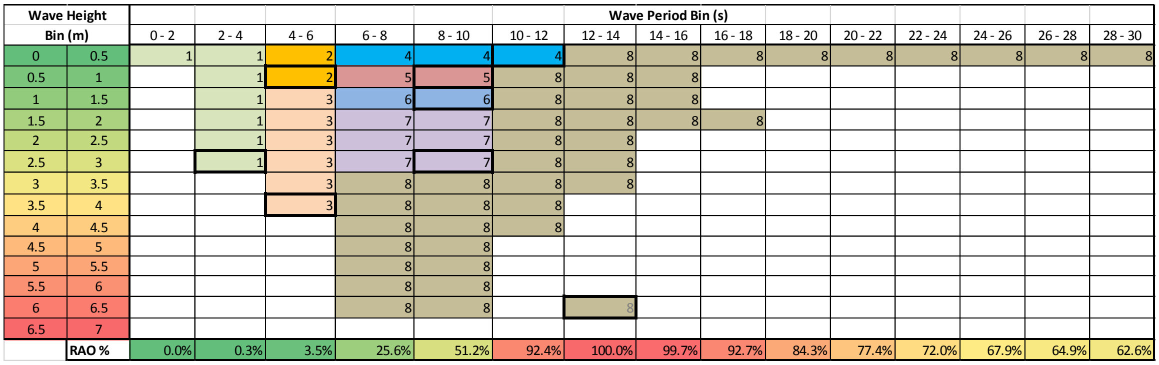
(H*RAO) * Nwaves %

Wave Height		Wave Period Bin (s)														
Bin (m)		0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20	20 - 22	22 - 24	24 - 26	26 - 28	28 - 30
0	0.5	0.0%	0.9%	8.3%	10.4%	1.6%	0.3%	0.4%	0.1%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%	0.0%
0.5	1	--	0.3%	8.9%	20.2%	5.1%	0.7%	0.1%	0.0%	--	--	--	--	--	--	--
1	1.5	--	0.0%	3.4%	13.9%	4.4%	0.3%	0.1%	0.0%	--	--	--	--	--	--	--
1.5	2	--	0.0%	1.1%	7.5%	2.6%	0.1%	0.0%	0.0%	0.0%	--	--	--	--	--	--
2	2.5	--	0.0%	0.3%	3.7%	1.2%	0.1%	0.0%	--	--	--	--	--	--	--	--
2.5	3	--	0.0%	0.1%	1.8%	0.4%	0.0%	0.0%	--	--	--	--	--	--	--	--
3	3.5	--	--	0.0%	0.8%	0.1%	0.0%	0.0%	--	--	--	--	--	--	--	--
3.5	4	--	--	0.0%	0.4%	0.0%	0.0%	--	--	--	--	--	--	--	--	--
4	4.5	--	--	--	0.1%	0.0%	0.0%	--	--	--	--	--	--	--	--	--
4.5	5	--	--	--	0.0%	0.0%	--	--	--	--	--	--	--	--	--	--
5	5.5	--	--	--	0.0%	0.0%	--	--	--	--	--	--	--	--	--	--
5.5	6	--	--	--	0.0%	0.0%	--	--	--	--	--	--	--	--	--	--
6	6.5	--	--	--	0.0%	0.0%	--	--	--	--	--	--	--	--	--	--
6.5	7	--	--	--	--	--	--	--	--	--	--	--	--	--	--	--
	RAO %	0.0%	0.3%	3.5%	25.6%	51.2%	92.4%	100.0%	99.7%	92.7%	84.3%	77.4%	72.0%	67.9%	64.9%	62.6%

Figure 13: Scatter table of estimated relative fatigue damage

Figure 14 shows the scatter table reduced to 8 blocks. Bold outline denotes a master wave for a block. The master wave has the largest wave height and RAO value for the block. The estimate of relative fatigue damage is computed for each block using Eq. 5, the master wave height, associated RAO value and summed occurrences. The percentage estimate of a block is limited (e.g. < 20%).

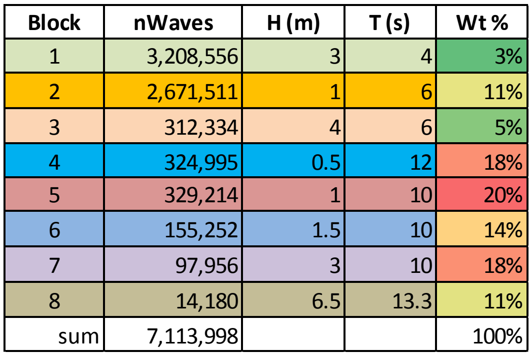
The procedure still requires some subjectivity in selecting the blocks. An example here is the isolated master wave in block 8. This is a fictitious wave assigned with one occurrence. This master wave allows larger coverage for block 8 and avoids an additional two extra blocks with tiny amounts of estimated relative fatigue damage.



Wave Height		Wave Period Bin (s)														
Bin (m)		0 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 20	20 - 22	22 - 24	24 - 26	26 - 28	28 - 30
0	0.5	1	1	2	4	4	4	8	8	8	8	8	8	8	8	8
0.5	1		1	2	5	5	8	8	8							
1	1.5		1	3	6	6	8	8	8							
1.5	2		1	3	7	7	8	8	8	8						
2	2.5		1	3	7	7	8	8								
2.5	3		1	3	7	7	8	8								
3	3.5			3	8	8	8	8								
3.5	4			3	8	8	8									
4	4.5				8	8	8									
4.5	5				8	8										
5	5.5				8	8										
5.5	6				8	8										
6	6.5				8	8		8								
6.5	7															
	RAO %	0.0%	0.3%	3.5%	25.6%	51.2%	92.4%	100.0%	99.7%	92.7%	84.3%	77.4%	72.0%	67.9%	64.9%	62.6%

Figure 14: Scatter table condensed to 8 blocks

Figure 15 shows for each block, the number of occurrences (nWaves), wave height (H), wave period (T) and estimate of relative fatigue damage (Wt). It is not possible to size the blocks with equal Wt. Shifting a master wave to increase the size of a block can produce an excessive increase in Wt, (e.g. from 5 to 50 %), especially with an increase in in the RAO value. This causes some bins to have smaller Wt values.



Block	nWaves	H (m)	T (s)	Wt %
1	3,208,556	3	4	3%
2	2,671,511	1	6	11%
3	312,334	4	6	5%
4	324,995	0.5	12	18%
5	329,214	1	10	20%
6	155,252	1.5	10	14%
7	97,956	3	10	18%
8	14,180	6.5	13.3	11%
sum	7,113,998			100%

Figure 15: Particulars of each wave block

8. Short Crested Seas – Fatigue Waves

Moderate sea states often comprise short crested waves and are less readily converted to long crested design waves. The amount of available measurement and design data available for short crested seas is also limited. There is however a growing interest with including short crested wave data in fatigue design for flexible risers.

This section outlines a method for converting scatter tables of short-crested waves to an equivalent scatter table of long crested waves, specifically for fatigue analysis of risers. Platform heading and riser hang-off heave displacement are used in conversion process.

A short crested sea often comprises independent wave systems travelling in different directions, for example swell and wind waves originating from separate storm centres. Directional wave spreading may also contribute to the interaction effect of the two wave systems, although this process is not addressed here. Wave measurement data for short crested seas can be resolved as two independent long crested wave spectra with different directions of travel. An FPSO heading is a third direction of significance, as determined from mooring or station keeping design with wind, current and wave loading.

Fatigue analysis of flexible risers is based on long crested regular wave cycles. Local analysis of armour fatigue in particular uses regular cycles of bending curvature or bending via tension-angle loading. Loading in local analysis is applied in a single plane, which conservatively removes loading directionality from the analysis. Local analysis is vendor specific and not readily adapted for more generalised types of loading. The onus is to convert short crested wave data to a format that is compatible with present flexible riser design practice.

A procedure for converting a short crested sea state to equivalent long crested regular waves for fatigue analysis of flexible risers is outlined as follows.

A short crested sea state is resolved as two independent long crested sea states, for example two JONSWAP spectra with independent parameters and directions. The spectra is converted to independent scatter tables for occurrences of individual waves (normalized to percentage occurrence). The wave data would normally be supplied as basis information for a riser design. FPSO heading in each short crested sea should preferably be available for the coexisting wind, current and wave condition.

Riser hang-off heave motion is used for removing directionality from the loading conditions. The individual wave heights and periods in a scatter table are converted to a scatter table of riser hang-off heave amplitudes. The most applicable RAO for riser hang-off heave is used. The RAO should account for directionality of each wave scatter table and platform heading. A heave scatter table is prepared for each wave scatter table. The formulation for this step is summarized as

$h_{1ij} = H_{1i} \text{ x } RAO(\theta_1, T_{1j})$ and $h_{2mn} = H_{2m} \text{ x } RAO(\theta_2, T_{2n})$ (Eq. 6)

where h denotes riser hang-off heave displacement (double amplitude), H wave height, T wave period, θ wave direction relative to the floating platform, (1, i, j) and (2, m, n) indices for tables (1, 2).

Directionality is removed by converting each heave scatter table to wave heights incident from a common direction. The formulation for this step is summarized as

$H'_{1ij} = h_{1ij} / RAO(\theta_0, T_{1j})$ and $H'_{2mn} = h_{2mm} / RAO(\theta_0, T_{2n})$ (Eq. 7)

where H' denotes modified wave height and θ_0 a common wave direction, e.g. head waves.

The wave scatter tables with percentage occurrences (p_{1ij}, p_{2mn}) have been augmented with tables of modified heights (H'_{1ij}, H'_{2mn}) where

Table 1 = (H_{1i}, T_{1j}, p_{1ij}, H'_{1ij}) and Table 2 = (H_{2m}, T_{2n}, p_{2mn}, H'_{2mn}).

The procedure now combines the two tables. Individual waves in the two tables (component sea states) are assumed independent. The joint probability of occurrence is the product of individual probabilities (percentage occurrences)

$p_{3ijmn} = p_{1ij} \text{ x } p_{2mn}$ (Eq. 8)

where p_3 denotes the combined probability of occurrence. Individual wave heights are combined in accordance with energy

$H_{3ijmn} = ((H'_{1ij})^2 + (H'_{2mn})^2)^{1/2}$. (Eq. 9)

Individual wave periods are combined via an average wave frequency weighted by wave heights

$f_{3ijmn} = (H'_{1ij} \times f_{1j} + H'_{2mn} \times f_{2n}) / (H'_{1ij} + H'_{2mn})$ (Eq. 10)

where frequency f = 1/T. An alternative weighting is wave height squared in accordance with wave energy. The weighting is empirically based and ensures the period of an individual wave with large height is not significantly modified when combined with an individual wave of small height. The combined wave period is representative of the resulting individual wave rather than a superposition of two regular wave trains.

The combined scatter table for individual waves occurrences (H_{3ijmn}, T_{3ijmn}, p_{3ijmn}) has several more entries than the component tables (H_{1i}, T_{1j}, p_{1ij}) and (H_{2m}, T_{2n}, p_{2mn}). For example if Tables 1 and 2 have 20 entries each then the (combined) Table 3 has 400 entries.

A final table is produced by allocating the entries in Table 3 to wave height and period bins comparable to those in the independent scatter tables (i.e. Tables 1, 2). The formulation of this step is summarised as

$p_{4rs} = \Sigma(p_{3ijmn})$ for $H_{4,r-1} < H_{3ijmn} \leq H_{4r}$ and $T_{4,s-1} < T_{3ijmn} \leq T_{4s}$ (Eq. 11)

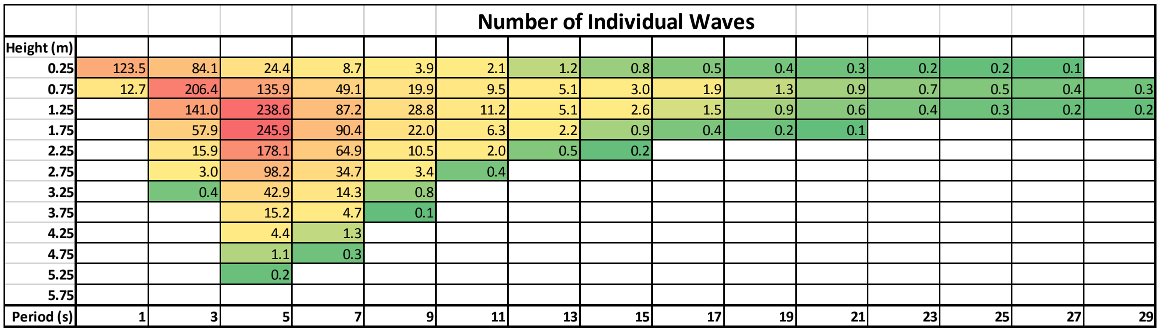
where ($H_{4,r-1}$, H_{4r}) and ($T_{4,s-1}$ T_{4s}) denote the wave height and period bins in Table 4 and p_{4rs} are the summed probabilities of occurrence. The probabilities are converted to number of occurrences by a scale factor so that the total duration of the waves equals a required nominal value, e.g. three hours.

Figure 16 to Figure 18 show an example of component wave scatter tables for a short crested sea and the combined scatter table developed for fatigue analysis of flexible risers. The wave heights and periods bins are shown with midpoint values, (e.g. height bin 0 to 0.5 m shown as 0.25 m, period bin 0.0 to 2.0 s shown as 1.0 s).

The combined scatter table has a similar distribution of wave heights and periods as Scatter Table 1. There is a small redistribution of wave occurrences towards period bins 3 and 5 s due to interaction with Scatter Table 2. The total number of wave occurrences is approximately midway between Scatter Tables 1 and 2, (20 percent higher than Scatter Table 1).

The process is repeated for several short crested sea scatter tables and the combined scatter tables summed. The final combined scatter table is then reduced to a smaller number of blocks as described in Section 7.

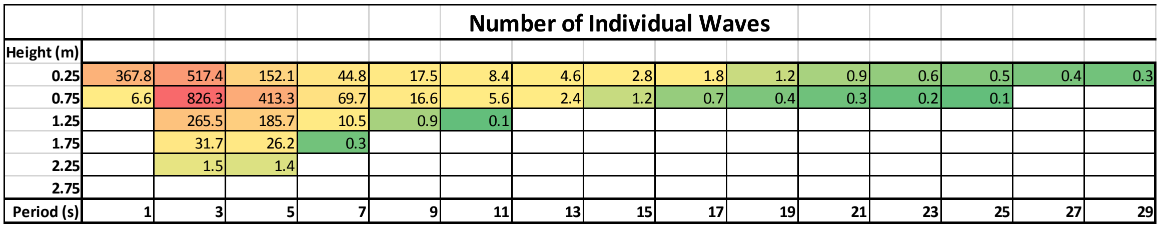
Hs 2.4 m, Tp 6.8 s, from North, No. waves 2141 per 3 hr



							Number of Individual Waves								
Height (m)															
0.25	123.5	84.1	24.4	8.7	3.9	2.1	1.2	0.8	0.5	0.4	0.3	0.2	0.2	0.1	
0.75	12.7	206.4	135.9	49.1	19.9	9.5	5.1	3.0	1.9	1.3	0.9	0.7	0.5	0.4	0.3
1.25		141.0	238.6	87.2	28.8	11.2	5.1	2.6	1.5	0.9	0.6	0.4	0.3	0.2	0.2
1.75		57.9	245.9	90.4	22.0	6.3	2.2	0.9	0.4	0.2	0.1				
2.25		15.9	178.1	64.9	10.5	2.0	0.5	0.2							
2.75		3.0	98.2	34.7	3.4	0.4									
3.25		0.4	42.9	14.3	0.8										
3.75			15.2	4.7	0.1										
4.25			4.4	1.3											
4.75			1.1	0.3											
5.25			0.2												
5.75															
Period (s)	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**	**21**	**23**	**25**	**27**	**29**

Figure 16: Wave scatter table of the swell – Scatter Table 1

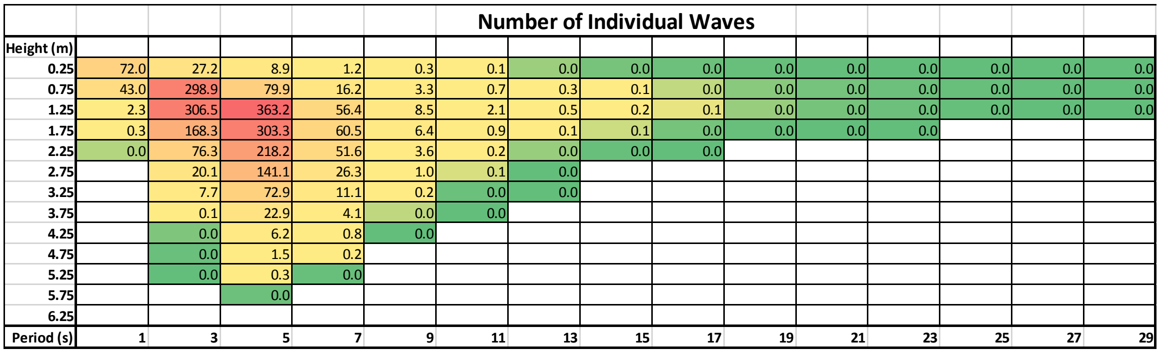
Hs 1.0 m, Tp 5.0 s, from North East, No. waves 2988 per 3 hr



							Number of Individual Waves								
Height (m)															
0.25	367.8	517.4	152.1	44.8	17.5	8.4	4.6	2.8	1.8	1.2	0.9	0.6	0.5	0.4	0.3
0.75	6.6	826.3	413.3	69.7	16.6	5.6	2.4	1.2	0.7	0.4	0.3	0.2	0.1		
1.25		265.5	185.7	10.5	0.9	0.1									
1.75		31.7	26.2	0.3											
2.25		1.5	1.4												
2.75															
Period (s)	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**	**21**	**23**	**25**	**27**	**29**

Figure 17: Wave scatter table of the wind sea – Scatter Table 2

Combined Scatter Table, No. waves 2498 per 3 hr



							Number of Individual Waves								
Height (m)															
0.25	72.0	27.2	8.9	1.2	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
0.75	43.0	298.9	79.9	16.2	3.3	0.7	0.3	0.1	0.0	0.0	0.0	0.0	0.0	0.0	0.0
1.25	2.3	306.5	363.2	56.4	8.5	2.1	0.5	0.2	0.1	0.0	0.0	0.0	0.0	0.0	0.0
1.75	0.3	168.3	303.3	60.5	6.4	0.9	0.1	0.1	0.0	0.0	0.0	0.0			
2.25	0.0	76.3	218.2	51.6	3.6	0.2	0.0	0.0	0.0						
2.75		20.1	141.1	26.3	1.0	0.1	0.0								
3.25		7.7	72.9	11.1	0.2	0.0	0.0								
3.75		0.1	22.9	4.1	0.0	0.0									
4.25		0.0	6.2	0.8	0.0										
4.75		0.0	1.5	0.2											
5.25		0.0	0.3	0.0											
5.75			0.0												
6.25															
Period (s)	**1**	**3**	**5**	**7**	**9**	**11**	**13**	**15**	**17**	**19**	**21**	**23**	**25**	**27**	**29**

Figure 18: Wave scatter table of short crested waves – for riser analysis

9. Conclusions

Heave motion at a flexible riser hang-off on a floating platform can be effectively used for design. Conclusions in the three areas of application presented in this paper are outlined as follows.

Extreme Waves

Flexible riser hang-off heave motion provides an effective measure for selecting design waves. Periods of a design regular wave can be selected to ensure the most probable maximum wave height produces the most probable maximum of riser hang-off heave amplitude (displacement, velocity or acceleration). The wave period selection is a formalised method that avoids overly conservative heave amplitudes at the riser hang-off.

The selected value of the wave period depends on if the heave motion is a displacement, velocity or acceleration. The resulting wave periods are generally near the spectral-peak period (Tp), which is in-line with expectation. A flexible riser design should generally assess system responses for the most probable maximum of hang-off heave displacement, velocity and acceleration.

Fatigue Waves

Individual wave scatter tables can be reduced to a small number of blocks with more reliability and efficiency than presently practiced. The methodology described converts occurrences in a wave scatter table to estimates of relative fatigue damage contribution. These estimates provide the starting point for grouping wave bins in blocks. The estimates are iteratively revaluated as the grouping proceeds. Setting an upper limit on the estimated fatigue contribution of a block avoids potential for later subdivision of the blocks and avoids unnecessary delays.

Short-Crested Seas – Fatigue Waves

An accurate representation of short crested seas is difficult to include in flexible riser fatigue design and there are no known established practices. The design process for flexible risers is very dependent on regular wave loading for global and local analyses. The methodology described converts bi-directional sea states that comprise a short crested sea to a scatter table of individual wave occurrences. The method provides regular wave data for conventional fatigue analysis of flexible risers.